Low order regularity for elliptic and parabolic problems

In this talk we present partial regularity results for elliptic and parabolic problems admitting only very weak regularity properties in the coefficients. In the first part of the talk we consider vector valued minimizers $u \in W^{1,p}(\Omega, \mathbb{R}^N)$ of degenerate quasi-convex integral functionals of the type

$$\mathcal{F}[u] := \int_\Omega f(x, u, Du).$$

The crucial point here is that the integrand f admits very weak regularity properties. With respect to the gradient variable it satisfies degenerate / singular p-growth conditions without necessarily possessing a quasi-diagonal Uhlenbeck type structure, and with respect to the x-variable the integrand might be even discontinuous. It is only assumed that a certain VMO-condition holds. Under those assumptions we prove partial Hölder-continuity of minimizers, i.e. Hölder continuity of u for any Hölder exponent $\alpha \in (0, 1)$ outside a set of measure zero. Under such weak assumptions regularity results for the gradient of minimizers is not expected to hold since even in the scalar case counterexamples to C^1-regularity are known.

The second part of the talk is concerned with parabolic systems of the form

$$\partial_t u = a(x, t, u, Du),$$

where the coefficients a admit p-growth conditions. Under the only assumption of continuous coefficients, i.e. $(x, t, u) \mapsto a(x, t, u, Du)$ is continuous, we obtain a partial Hölder continuity result for solutions. A key component throughout the argument is the use of DiBenedetto's intrinsic geometry to accommodate the inhomogeneity in the system. A main technical point is that, although we are proving the Hölder continuity of solutions, we employ the intrinsic geometry using cylinders stretched according to the size of the spatial gradient Du, in an iteration scheme that does not necessarily imply the boundedness of the spatial gradient itself.