

Characterization of ellipses as uniformly dense domains with respect to a family of convex sets

Abstract: Given $K \subset \mathbb{R}^N$ a convex body containing the origin, a measurable set $G \subset \mathbb{R}^N$ with positive Lebesgue measure is said to be uniformly K -dense if the measure of the sets $G \cap (x + r K)$ is constant on the boundary of G , for any fixed $r > 0$. We show that, for $N = 2$, G is uniformly K -dense if and only if K and G are homothetic ellipses and we discuss some properties of such sets in higher dimension.