THE REGULARITY OF GENERAL PARABOLIC SYSTEMS WITH DEGENERATE DIFFUSION

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ABSTRACT. The aim of the talk is twofold. On one hand we want to present a new technique called p-caloric approximation, which is a proper generalization of the classical compactness methods first developed by DeGiorgi with his Harmonic Approximation Lemma. This last result, initially introduced in the setting of Geometric Measure Theory to prove the regularity of minimal surfaces, is nowadays a classical tool to prove linearization and regularity results for vectorial problems. Here we develop a very far reaching version of this general principle devised to linearize general degenerate parabolic systems. The use of this result in turn allows to achieve the subsequent and main aim of the paper, that is the implementation of a partial regularity theory for parabolic systems with degenerate diffusion of the type

$$\partial_t u - \operatorname{div} a(Du) = 0\,, \tag{0.1}$$

without necessarily assuming a quasi-diagonal structure, i.e. a structure prescribing that the gradient non-linearities depend only on the explicit scalar quantity $|Du|$. Indeed, the by now classical theory of DiBenedetto introduces the fundamental concept of intrinsic geometry and allows to deal with the classical degenerate parabolic p-Laplacean system

$$\partial_t u - \operatorname{div}(|Du|^{p-2}Du) = 0 \tag{0.2}$$

and more general with systems of the type

$$\partial_t u - \operatorname{div}\big(g(|Du|)Du\big) = 0\,. \tag{0.3}$$

Here, we take such regularity results as a starting point and develop a partial regularity theory – regularity of solutions outside a negligible closed subset of the domain – applying to general degenerate parabolic systems of the type (0.1), thereby not necessarily satisfying a structure assumption as (0.3). The partial regularity rather than the everywhere one, is natural since even in the non-degenerate case, when considering systems with general structure, singularities may occur. The proof of the almost everywhere regularity of solutions is then achieved via an extremely delicate combination of local linearization methods, together with a proper use of DiBenedetto's intrinsic geometry: the general approach that consists in performing the local analysis by considering parabolic cylinders whose space-time scaling depend on the local behavior of the solution itself. The combination of these approaches was exactly the missing link to prove partial regularity for general parabolic systems considered in (0.1). In turn, the implementation realizing such a matching between the two existing theories is made possible by the p-caloric approximation lemma. More precisely, the proof involves two different kinds of linearization techniques: a more traditional one in those zones where the system is non-degenerate and the original solution is locally compared to solutions of a suitable linear system, and a degenerate one in the zones where the system is truly degenerate and the solution can be compared with solutions of systems as (0.2) via the p-caloric approximation lemma.

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