We investigate the regularity of the weak solutions for degenerate elliptic equations of the following kind

$$\text{div}\, A(x, u(x), \nabla u(x)) + B(x, u(x), \nabla u(x)) = 0\,, \tag{1}$$

under the structure conditions

$$\begin{cases} |A(x, u, \xi)| \le a\omega(x)|\xi|^{p-1} + b|u|^{p-1} + e \\ |B(x, u, \xi)| \le c|\xi|^{p-1} + d|u|^{p-1} + f \\ \xi \cdot A(x, u, \xi) \ge \omega(x)|\xi|^p - d|u|^p - g\,. \end{cases} \tag{2}$$

Such equations have been studied by many Authors in the case $\omega(x) \equiv 1$ or ω an A_2 Muckenhoupt weight. Here v is a strong A_∞ weight and $\omega = v^{1-\frac{p}{n}}$, $1 < p < n$.

The novelty here is the degeneracy condition given by choice of the weight ω to be a power of a strong A_∞ weight. Moreover, we assume very mild integrability conditions on the lower order terms. These conditions are sharp and - at least in some instances - are necessary and sufficient.

Our main result is the Harnack inequality for nonnegative weak solutions. As a direct consequence, smoothness for weak solutions is obtained. In particular, we have continuity result under Stummel - Kato type assumptions and hölder continuity result under Morrey type assumptions.