Higher differentiability and regularity for minimizers of convex multiple integrals

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We consider multiple integrals with convex integrand satisfying (p, q) growth conditions. Namely we deal with functionals of the form

$$\mathcal{I}(v, \Omega) = \int_\Omega F(Dv(x))\,\mathrm{d}x$$

where Ω is an open subset of $\mathbb{R}^n$ and $v\colon \Omega \to \mathbb{R}^N$ is a suitable map. Here we are concerned with the multi-dimensional cases n, $N \geq 2$ and with integrands $F\colon \mathbb{R}^{N\times n} \to \mathbb{R}$ satisfying, for exponents $q \geq p \geq 2$ and positive constants L, c, the following hypotheses:

$$0 \leq F(\xi) \leq L(1 + |\xi|^q) \tag{H1}$$

and

$$\langle D^2F(\xi)\eta, \eta\rangle \geq c(1 + |\xi|^2)^{\frac{p-2}{2}}|\eta|^2 \tag{H2}$$

for all ξ, $\eta \in \mathbb{R}^{N\times n}$.

We prove higher differentiability results for bounded local minimizers of the functional $\mathcal{I}$, and for not a priori bounded local minimizers in the case $p \geq n - 2$, under dimensionless conditions on the gap between the growth and coercivity exponents.