A GEOMETRIC APPROACH TO SOME OVERDETERMINED PROBLEMS IN POTENTIAL THEORY

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In this talk, we present a method which we have developed in order to deal with some overdetermined elliptic boundary value problems. We illustrate this approach for a classical problem arising in potential theory in the exterior domain $\mathbb{R}^n \setminus \overline{\Omega}$: the one which defines the electrostatic potential u. The main idea is to reformulate the problem in terms of a metric g which is conformally equivalent to the flat metric by a factor defined as a suitable power of u. This reformulation allows us to identify a natural boundary condition on the mean curvature of $\partial\Omega$ under which the original problem becomes overdetermined and which ensures the splitting of the metric g as a Riemannian product. By the conformally flatness of g, we can then deduce that Ω is a ball and that u is rotationally symmetric.