Affine vs. Euclidean Isoperimetric Inequalities

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The equivalence of the classical Euclidean isoperimetric inequality and the sharp L_1 Sobolev inequality is a beautiful example of the interplay between geometric and analytic inequalities. This remarkable link has been amplified by the recent work on *affine* isoperimetric and analytic inequalities. An *affine isoperimetric inequality* is an inequality between geometric functionals which are invariant under volume preserving linear transformations. In contrast to the common misbelief that Euclidean inequalities (that is, inequalities for functionals invariant under rigid motions only) are stronger than affine inequalities, it has become apparent over the last decades that affine isoperimetric inequalities are the most powerful inequalities of (convex) geometric analysis.

In this talk we discuss, in particular, one of the most important examples of an affine isoperimetric inequality: *Petty's projection inequality.* It states that among compact, convex sets of given volume, the ones whose polar projection bodies have maximal volume are precisely the ellipsoids. We will give an overview of the impact over the last decade of this inequality, it's analytic version (the *affine Zhang-Sobolev inequality*), and their L_p generalizations by Lutwak-Yang-Zhang and Haberl and myself. We will also shed new light on the special role of Petty's projection inequality in affine geometric analysis by exploiting the fascinating connections between the theory of isoperimetric inequalities and the theory of valuations which were first discovered by Ludwig.