

Quasi-isospectral systems

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We show that if we start from one longitudinally vibrating rod we may construct one-parameter t -families of rods which have the same spectrum as the original rod, with the exception of a single eigenvalue which is free to move under some conditions on the parameter t . We call these rods *quasi-isospectral rods*.

The analysis uses the results obtained in [1] and [2], and it is essentially based on a double application of a Darboux Lemma [3] after reducing the equation to normal form by the Liouville transformation. The new rods and their normal modes can be constructed explicitly by means of closed-form expressions.

We also show how to use the above results to construct a rod under given end conditions with prescribed values of a finite set of eigenvalues. The reconstruction procedure needs the specification of an initial rod whose eigenvalues must be close to the assigned eigenvalues.

The analysis can be adapted to construct quasi-isospectral strings and some special classes of quasi-isospectral beams in bending vibration [4].

References

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