What's So Special about Minkowski Addition?

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ABSTRACT

One of the most widely applied operations in mathematics is vector addition. It can be used to define an operation between sets: Given two sets K and L, one defines K + L to be the set of all vector sums x + y, where x is in K and y is in L. As an operation between sets, vector addition is usually called Minkowski addition, and is the single most important set operation in geometry. There are several other wellknown and useful operations between sets in geometry—for example, L_p addition, radial addition, and Blaschke addition—but surprisingly few. Why is this? What is so special about the known operations, Minkowski addition in particular?

The talk is a report on a long-term project, still in progress, with Daniel Hug and Wolfgang Weil of the University of Karlsruhe. The general goal is to achieve a proper understanding of the fundamental nature of operations between sets in geometry. We have been able to prove some surprising theorems that shed light on the question in the title.