WEAK SOLUTIONS TO THE HEAT FLOW FOR SURFACES OF PRESCRIBED MEAN CURVATURE

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ABSTRACT. In this talk we establish the existence of global weak solutions to the heat flow for surfaces of prescribed mean curvature, i.e. the existence for the Cauchy-Dirichlet problem to parabolic systems of the type

$$\begin{cases} \partial_t u - \Delta u = -2(H \circ u) D_1 u \times D_2 u & \text{in } B \times (0,\infty), \\ u = u_o & \text{on } \partial_{\rm par}\big(B \times (0,\infty)\big), \end{cases}$$

where $H\colon \mathbb{R}^3 \to R$ is a bounded continuous function satisfying an isoperimetric condition, B the unit ball in $\mathbb{R}^2$ and $u\colon B \times (0,\infty) \to \mathbb{R}^3$. As one of the possible applications we show that the problem has a solution with values in $B_R \subset \mathbb{R}^3$, whenever $u_o(B) \subseteq B_R$ and furthermore there holds

$$\int_{\{\xi \in B_R : |H(\xi)| \geq \frac{3}{2R}\}} |H|^3 \, d\xi < \frac{9\pi}{2}, \qquad |H(a)| \leq \tfrac{1}{R} \quad \text{for } a \in \partial B_R.$$

The results that will be presented in the talk are joint work with Verena Bögelein und Chrsitoph Scheven from Erlangen.

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