OPTIMAL SOLVABILITY FOR FRACTIONAL p-LAPLACIAN EQUATIONS

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Abstract

We study existence and unuqueness of the solution to the following nonlocal Dirichlet problem:

$$\begin{cases} (-\Delta)_p^s u = f(x,u) & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{in } \Omega^c, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p > 1$, $s \in (0,1)$, and the leading operator is the fractional p-Laplacian defined by

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \to 0^+} \int_{B_\varepsilon^c(x)} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}}\, dy.$$

The reaction $f : \Omega \times \mathbb{R}^+ \to \mathbb{R}$ is subject to a one-sided growth condition and a monotonicity condition:

$$f(x,t) \leqslant c_0(1+t^{p-1}),\ t \mapsto \frac{f(x,t)}{t^{p-1}} \text{ decreasing in } (0,\infty).$$

We detect a necessary and sufficient condition for problem (1) to have a solution, and in addition uniqueness of such solution. Before giving the proof of our result in detail, we will review some well-known results for both local [2,3] and nonlocal problems [1,5]. We will especially focus on the novel difficulties of the nonlocal, nonlinear framework and how to solve them.

Work in collaboration with D. Mugnai [4].

References

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