Speaker: Richard Gardner, Western Washington University.

Title: The Minkowski Problem.

Abstract: It is a curious fact that the outer normal vectors to the facets of a convex polytope in n-dimensional Euclidean space $\mathbb{R}^n$, each with length equal to the area (i.e., $(n-1)$-dimensional volume) of the corresponding facet, sum up to the zero vector. There is a neat trick to see this in the plane: Just put the vectors end to tail and rotate by $\pi/2$. It is not so easy in higher dimensions and much more challenging still to prove the converse, that given *any* finite set of vectors that span $\mathbb{R}^n$ and whose sum is the zero vector, there is a unique (up to translation) convex polytope whose facet-area-weighted outer normal vectors are precisely the given ones. This is Minkowski's existence theorem, published by the great Hermann Minkowski in 1897. As is so often the case in mathematics, it represents the beginning of a story rather than the end.

The story is that of the Minkowski problem, and it has been unfolding for more than a century. Likened to the Rosetta Stone by the famous Italian mathematician Eugenio Calabi, the Minkowski problem is central not only to convex geometry, but other areas as well. Differential geometers think of it as asking for a convex surface with prescribed Gauss curvature, while mathematicians working in partial differential equations view it as requiring solutions to certain Monge-Ampère equations. The dozens of contributors include a Fields medalist and an Abel prizewinner.

The talk is aimed at a general audience. Using plenty of pictures, and avoiding technicalities as far as possible, I shall attempt to explain the Minkowski problem and its history up to the present day. There is much to say and there will only be a few minutes at the end for a very brief mention of my own recent work on the problem with collaborators Daniel Hug and Wolfgang Weil (Karlsruhe Institute of Technology), Sudan Xing (University of Alberta), and Deping Ye (Memorial University of Newfoundland).