KERNL3D, A Backprojection Kernel for Very-wide-angle 3D PET or TOF-PET Image Reconstruction

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Abstract

In KERNL3D we derive a kernel function $K(y_1, y_2, \varphi)$ whose backprojections from all directions (θ, φ) in the spherical band $|\varphi| < \bar{\varphi}_{\max}$ on the celestial sphere, when integrated with respect to solid angle over that band, yield the Gaussian point response function (PRF) $\rho = \exp(-(x^2+y^2+z^2)/2)$. This kernel is the analog, for large angle 3D tomography, of the kernel $H(t)$ for the Gaussian-blurred filtered backprojection method of 2D tomography. This K, when convolved against parallel-beam outprojection data $g(y_1, y_2, \theta, \varphi)$ from an unknown density function $f(x, y, z)$ for each direction (θ, φ) in the band, and the backprojections of the resulting convolutions $K{*}g$ are then integrated with respect to solid angle over that band, would yield the "mollification" $ff = \rho * f$, which is a slightly blurred version of f, and which stabilizes the mild ill-posedness of tomographic reconstruction. For PET (positron emission tomography) that backprojection reconstruction occurs stochastically and one emission event at a time. We derive the kernel K by a complicated averaging of rotated 2D backprojection kernels and discuss its use in backprojection, and needed data corrections. We describe Octave codes to tabulate the kernel K and to test its use in backprojection with a large angle $\bar{\varphi}_{\max} = \pi/3$. That K becomes independent of φ as $\bar{\varphi}_{\max}$ approaches $\pi/2$. The strengths of KERNL3D include using the PET data as it is actually collected, localness, simplicity, and the ability to count emission events in an isolated blob despite *very* low event counts. KERNL3DTOF retains these strengths and adds further advantages. It allows one to restrict backprojection to a spherical patch about the time-of-flight approximate location of each event. These "*backplacements*" help prevent noise and streaking in one region from contaminating the reconstruction in more distant regions. These strengths, as well as the increased geometric sensitivity from the large aperature $\bar{\varphi}_{\max}$, should combine with newer TOF–PET scanners of large axial length to allow *dynamic* collection of metabolic and chemical target information *simultaneously* from multiple organs such as brain, lungs, liver, ... Moreover, they should be useful in *detecting lesions* in clinical patients.