

Rogers-Shephard type inequalities and generalizations

Michael Roysdon– Department of Mathematical Sciences, Kent State University

Abstract. The study of volume of convex bodies is well developed and related to several areas of mathematics and computer science. The question of what happens if volume is replaced by an arbitrary measure on a convex body has not been considered in convex geometry until very recently, mostly because it is hard to believe that difficult geometric results can hold in such generality. However, in 2005 Zvavitch proved that the solution to the Busemann-Petty problem, the signature result in convex geometry, remains exactly the same if volume is replaced by an arbitrary measure with continuous density. Very recently, it was shown that several partial results on the slicing problem, a major open question in the area, can also be extended to arbitrary measures. Significant work has been done to explore the validity of the Brunn-Minkowski inequality for different classes of measures in place of volume.

In this context, it was recently discovered that the Rogers-Shephard inequality, a major tool in the study of non-symmetric convex bodies, which state that for an arbitrary convex body $K \subset R^n$

$$\text{vol}_n(K - K) \leq \binom{2n}{n} \text{vol}_n(K),$$

with equality only when K is an n -dimensional simplex. In this talk, we present a generalization of the Rogers-Shephard inequality to the setting of measures with radially decreasing densities, as well as, an extension of this result to the functional setting to the class of quasi-concave functions.
