MONOTONICITY AND SYMMETRY OF SOLUTIONS TO A NON-COOPERATIVE SYSTEM OF GROSS-PITAEVSKII-TYPE

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Abstract. The talk is devoted to the study of the qualitative properties of solutions to the elliptic system

$$(P)\qquad \begin{cases} -\Delta u = u - u^3 - \Lambda u v^2 & \text{in } \mathbb{R}^N \\ -\Delta v = v - v^3 - \Lambda u^2 v & \text{in } \mathbb{R}^N \\ u, v \geq 0 & \text{in } \mathbb{R}^N, \end{cases}$$

where $N \geq 1$ and $\Lambda > 1$.

This problem arises in the study of two-components Bose-Einstein condensates in the segregation regime (= coupling parameter $\Lambda > 1$).
In particular, we show a (sharp) universal L^∞ estimate for any solution of (P), as well as the monotonicty and the one-dimensional symmetry of any solution of (P) satisfying the asymptotic conditions

$$(h_\infty)\qquad \begin{aligned} &u(x', x_N) \to 1 \qquad v(x', x_N) \to 0 \qquad \text{as } x_N \to +\infty, \\ &u(x', x_N) \to 0 \qquad v(x', x_N) \to 1 \qquad \text{as } x_N \to -\infty. \end{aligned}$$

The talk is based on a joint work with Berardino Sciunzi and Nicola Soave.