An inverse problem of the equilibrium plasma in a tokamak and a corresponding extremal problem

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For suppression of plasma instabilities in a tokamak is necessary to know the distribution of electric current $j : \overline{\omega} \ni (x,y) \mapsto j(x,y) \geq 0$ flowing through the cross section ω plasma discharge. In the simplest case, $j(x,y) = f_u(x,y)$, where $f_u(x,y) = f\big(u(x,y)\big)$. What as regards the functions f and u, it is known only that

$$\Big| \int_\omega f\big(u(x,y)\big)\, dxdy - 1 \Big| \leq \mu \ll 1\,, \tag{1}$$

$$\Delta u = f\big(u(x,y)\big) \geq 0\,, \quad u\big|_{\gamma=\partial\omega} = 0, \quad \sup_{P\in\gamma} \Big| \frac{\partial u}{\partial \nu}(P) - \Phi(P) \Big| \leq \lambda \sup_{P\in\gamma} \Big| \Phi(P) \Big|\,. \tag{2}$$

Here $1/\lambda \gg 1$, and Φ is a given function. It has been shown [1] that for some Φ there are essentially different solutions f_u^1 and f_u^2 of the problem (1) – (2) in the sense that

$$\Big| \frac{\|f_u^1\| - \|f_u^2\|}{\max\{\, \|f_u^1\|\,,\, \|f_u^2\|\,\}} \Big| \geq \alpha \sim\ 0.1 \div 0.2\,, \quad \text{where} \quad \|f_u^j\| \stackrel{def}{=} \max_{(x,y)\in\omega} |f_u^j(x,y)|\,,$$

and

$$(\hat{x},\hat{y}) \in \text{absmax}\, f_u^1 \qquad \Rightarrow \qquad (\hat{x},\hat{y}) \in \text{absmin}\, f_u^2\,.$$

The report will consider the finding of **all** essentially different solutions to the problem (1)–(2).

[1] A. S. Demidov and V. V. Savelyev *Essentially different current distributions in the inverse problem for the Grad-Shafranov equation*, Russian Journal of Mathematical Physics (Impact Factor: 1.05) 03/2010.