

Optimal Gagliardo-Nirenberg type estimates

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Abstract.

Our main goal in this work is to study Gagliardo-Nirenberg type estimates of any order, involving arbitrary rearrangement invariant (r.i.) spaces. That is, we find necessary and sufficient conditions on $X(I^{n-1})$ and $Z(I^n)$ under which we have the Sobolev embeddings of the form

$$W^m Z(I^n) \hookrightarrow \mathcal{R}(X, L^\infty), \quad (1)$$

where $\mathcal{R}(X, L^\infty)$ is a mixed norm space. In particular, our analysis focuses on determining the optimal domains and the optimal ranges for (1) between r.i. spaces and mixed norm spaces. As a consequence, we prove that the classical embedding for the standard Sobolev space $W^1 L^p$ by Poornima and Peetre ($1 \leq p < n$), and by Hansson, Brézis, Wainger and Maz'ya ($p = n$) can be improved considering mixed norms as target spaces.

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