INdAM Meeting "Convex Geometry - Analytic Aspects"

Cortona, June 26th - 30th 2023



Picture by <u>anddm</u>

Book of Abstracts

Schedule

Monday June 26th

8:30-9:15	Registration	
9:15-9:30	Opening	
9:30-10:30	Bernig Andreas Frankfurt, Germany	Hermitian intrinsic volumes on Kaehler manifolds
10:30-11:00	Faifman Dmitry Tel Aviv, Israel	Funk geometry of polytopes and their flags
11:00-11:30	Coffee break	
11:30-12:30	Mussnig Fabian Vienna, Austria	An elementary question about functional intrinsic volumes
12:30-13:00	Li Jin Shanghai, China	The Legendre transform and dually epi-translation contravariant valuations
13:00-15:00	Lunch break	
15:00-15:30	Knoerr Jonas Vienna, Austria	Monge-Ampère operators and valuations
15:30-16:00	Hofstätter Georg Jena, Germany	Equivariant valuations on convex functions
16:00-16:30	Coffee break	
16:30-17:00	Semenov Vadim New York, US	The Uniqueness of the Gauss Image Measure
17:00-17:30	Mathis Léo Frankfurt, Germany	Expectation of random submanifold with zonoids

Tuesday June 27th

9:00-10:00	Artstein Shiri Tel Aviv, Israel	Vertex generated polytopes
10:00-10:30	Rotem Liran Haifa, Israel	Stability and equality cases for the Gaussian (B) inequality
10:30-11:00	Coffee break	
11:00-12:00	Livshyts Galyna Atlanta, US	An estimate for the Dimensional Brunn-Minkowski conjecture for all log-concave measures
12:00-12:30	Yepes Nicolás Jesús Murcia, Spain	On complemented Brunn-Minkowski type inequalities
13:00-15:00	Lunch break	
15:00-15:30	Li Ai-Jun Hangzhou, China	Some geometric and functional inequalities related to lower dimensional subspaces
15:30-16:00	Tatarko Kateryna Waterloo, Canada	Reverse isoperimetric problems under curvature constraints
16:00-16:30	Coffee break	
16:30-17:00	Besau Florian Vienna, Austria	Weighted floating bodies and polarity
17:00-17:30	Lin Youjian Chongqing, China	Projection bodies in spherical and hyperbolic spaces

Wednesday June 28th

9:00-10:00	Van Handel Ramon Princeton, US	Mixed area measures
10:00-10:30	Kotrbatý Jan Frankfurt, Germany	Inequalities for higher rank mixed volumes
10:30-11:00	Coffee break	
11:00-11:30	Hug Daniel Karlsruhe, Germany	Equality cases in the Alexandrov-Fenchel inequality for a class of convex bodies
11:30-12:00	Wannerer Thomas Jena, Germany	On a generalization of the Alexandrov-Fenchel inequality
12:00-12:30	Langharst Dylan Kent, US	Higher-Order Difference and Projection Bodies, and their associated Inequalities
12:30-13:00	Haddad Julián Seville, Spain	The L_p Higher-order Petty projection inequality
13:00	Lunch and free afternoon	

Thursday June 29th

9:00-10:00	Cianchi Andrea Florence, Italy	On a double-symmetrization inequality for anisotropic integrals of Sobolev functions
10:00-10:30	Koldobsky Alexander Columbia, US	Comparison theorems for the Radon transform
10:30-11:00	Coffee break	
11:00-12:00	Zhao Yiming Syracuse, US	The Minkowski problem in Gaussian probability space (online)
12:00-12:30	Saroglu Christos Ioannina, Greece	On a j-Santaló conjecture
13:0-15:00	Lunch break	
15:00-15:30	Paouris Grigoris College Station, US	Concentration of marginals of p-Schatten norms
15:30-16:00	Wyczesany Katarzyna Pittsburg, US	Stability of polydisc slicing
16:00-16:30	Coffee break	
16:30-17:00	Ortega-Moreno Oscar Vienna, Austria	Fixed Points of Mean Section Operators
17:00-17:30	Mui Stephanie New York, US	TBA

Friday June 30th

9:00-10:00	Böröczky Károly Budapest, Hungary	L_p -Minkowski problems - Old and new results
10:00-10:30	Ulivelli Jacopo Rome, Italy	Entire Monge-Ampère equations and weighted Minkowski problems
10:30-11:00	Coffee break	
11:00-12:00	Reitzner Matthias Osnabrück, Germany	Random Polytopes in Polytopes
12:00-12:30	Giannopoulos Apostolos Athens, Greece	Threshold for the expected measure of random polytopes
12:30-13:00	Schneider Rolf Freiburg, Germany	Expected valuations of random zonotopes
13:00	Lunch	

Lectures

Artstein, Shiri (Tel Aviv University, Israel)

Title: Vertex generated polytopes

Abstract: Some curious Brunn-Minkowski type theorems will be discussed and lead us to the study of classes of polytopes with special covering properties. Joint work with Tomer Falah and Boaz Slomka.

Bernig, Andreas (Goethe University Frankfurt, Germany)

Title: Hermitian intrinsic volumes on Kaehler manifolds

Abstract: The intrinsic volumes of a convex body are expressed by Steiner's formula as the (suitably normalized) coefficients of the volume of a tube. A differential geometric version was given by H. Weyl, who showed that the volume of a small tube around a submanifold with boundary in euclidean space is a polynomial in the radius. More importantly, the suitably normalized coefficients are intrinsic, i.e. they only depend on the inner geometry of the manifold and not on the embedding. In modern language, the intrinsic volumes of Riemannian manifolds are valuations on manifolds that behave naturally under isometric embeddings.

We prove an analogous statement for Kaehler manifolds: there is a family of valuations on Kaehler manifolds, called hermitian intrinsic volumes, that behave naturally under holomorphic isometric embeddings. This is joint work with J. Fu (Athens GA), G. Solanes (Barcelona) and T. Wannerer (Jena).

Böröczky, Károly (Rényi Institute of Mathematics, Budapest, Hungary)

Title: L_p-Minkowski problem - Old and New results

Abstract: Lutwak's L_p Minkowski problem as a Monge-Ampère equation on the (n-1)dimensional sphere for real p has been in the center of attention the last couple of decades. The talk surveys the state of art (like the recent resolution of the case p < -n by Guang, Li, Wang, or stability versions strengthening Brendle, Choi and Daskalopoulos' celebrated result about the uniqueness of the unit ball as a solution for the suitable equation for p > -n), and points out some major open problems.

Cianchi Andrea (University of Florence, Italy)

Title: On a double-symmetrization inequality for anisotropic integrals of Sobolev functions

Abstract: A new approach to an inequality for anisotropic Dirichlet-type integrals of the gradient of Sobolev functions, which involves a symmetrization of both trial functions and the integrand, is proposed. This yields a geometric insight into the relevant inequality and avoids approximation arguments via sequences of Steiner symmetrizations or polarizations, which are used in the available proofs. Importantly, it provides a point of departure for the characterization of the cases of equality, which is a delicate issue even in the classical Pólya-Szegő inequality for the standard Dirichlet integral. This is joint work with G. Bianchi and P. Gronchi.

Livshyts, Galyna (Georgia Institute of Technology, Atlanta, US)

Title: An estimate for the Dimensional Brunn-Minkowski conjecture for all log-concave measures

Abstract: We will show that for any even log-concave measure μ and any pair of symmetric convex sets K and L, and any t between 0 and 1, one has the inequality: $\mu(tK+(1-t)L)^{c(n)} \ge t\mu(K)^{c(n)}+(1-t)\mu(L)^{c(n)}$, where $c(n) = n^{-4-o(1)}$. This constitutes progress towards the Dimensional Brunn-Minkowski conjecture.

Mussnig, Fabian (TU Wien, Austria)

Title: An elementary question about functional intrinsic volumes

Abstract: It is an almost trivial observation that if a convex body $K \in \mathcal{K}^n$ is such that its surface area vanishes, then also its volume vanishes. In fact, we can even say more, namely that the dimension of K is at most n - 2. We will present an analog of this statement on convex functions, and we will see that the situation is less trivial in this setting. Among others, this problem is related to new Cauchy–Kubota formulas for mixed Monge–Ampère measures.

Joint work with Daniel Hug and Jacopo Ulivelli.

Reitzner, Matthias (University of Osnabrück, Germany)

Title: Random Polytopes in Polytopes

Abstract: Let P be a d-dimensional polytope. Choose n uniform random points in P and let P_n be the convex hull of these random points. We are interested in the expectation of the number of vertices and the volume of P_n . The asymptotic behaviour of these functionals as n tends to infinity has been determined by Renyi and Sulanke 60 years ago in the planar case, results in general dimensions, on the variance and central limit theorems are more recent.

We will present classical results and recent developments and point out some open questions.

Van Handel, Ramon (Princeton University, US)

Title: Mixed area measures

Abstract: Much of the foundation for convex geometry was laid in Minkowski's seminal 1903 paper. Even now, on its 120th birthday, some of the most basic questions about convex bodies that arise from this work remain unresolved. My aim in this talk is to discuss some recent progress and problems surrounding one of these questions: the structure of mixed area measures, or, in the form studied by Minkowski, the equality cases of the monotonicity of mixed volumes. Based on joint work with Shouda Wang.

Zhao, Yiming (Syracuse University, US)

Title: The Minkowski problem in Gaussian probability space

Abstract: The classical Minkowski problem, which asks for the characterization of surface area measure in Euclidean space with Lebesgue measure, largely motivated the development of elliptic PDEs throughout the last century. In this talk, we will discuss the corresponding problem in Gaussian probability space. Lack of homogeneity and translation invariance make this problem fundamentally different from the classical problem. We will discuss uniqueness results as well as existence results. This is based on joint works with Yong Huang, Dongmeng Xi, and with Shibing Chen, Shengnan Hu, Weiru Liu.

Short talks

Besau, Florian (TU Wien, Austria)

Title: Weighted Floating Bodies and Polarity

Abstract: Meyer & Werner showed that Lutwak's *p*-affine surface area in *d*-dimensional Euclidean space arises as the volume derivative of the floating body of convex body conjugated by polarity for p = -d/(d+2). We establish an extension of this relation that allows us to replace the uniform Euclidean volume by a arbitrary non-uniform measure with a continuous density. As an immediate application we determine a natural analog to Lutwak's *p*-affine surface area in spherical and hyperbolic space. Based on joint work with E. Werner.

Faifman, Dmitry (Tel Aviv University, Israel)

Title: Funk geometry of polytopes and their flags

Abstract: Funk geometry is a lesser known cousin of the Hilbert geometry in the interior of a convex body. As it turns out, it brings together and generalizes a surprising amount of notions and questions from convex geometry. We will consider the Holmes-Thompson volume of metric balls in Funk geometry. By extremizing the latter for fixed radius, wededuce generalized Blaschke-Santalo and Mahler inequalities for unconditional bodies. We will then study the large radius asymptotics of the volume in the case of a polytope, which turn out to be intimately related to its flags. In particular, we will deduce the Kalai flag number conjecture for unconditional convex bodies, and recover a natural notion of centro-affine surface area adapted to polytopes. Based on joint work with C. Vernicos and C. Walsh.

Giannopoulos, Apostolos (National Technical University of Athens, Greece)

Title: Threshold for the expected measure of random polytopes

Abstract: We discuss the question how to obtain a threshold for the expected measure of a random polytope defined as the convex hull of independent random points with a log-concave distribution. For a precise formulation of the problem, let μ be a log-concave probability measure on \mathbb{R}^n and for any N > n consider the random polytope $K_N =$ $\operatorname{conv}\{X_1, \ldots, X_N\}$, where X_1, X_2, \ldots are independent random points in \mathbb{R}^n distributed according to μ . The question is if there exists a threshold for the expected measure $\mathbb{E}_{\mu^N}[\mu(K_N)]$ of K_N . Our approach is based on the Cramer transform Λ^*_{μ} of μ . We examine the existence of moments of all orders for Λ^*_{μ} and establish, under some conditions, a sharp threshold for $\mathbb{E}_{\mu^N}[\mu(K_N)]$: It is close to 0 if $\ln N \leq (1 - o_n(1))\mathbb{E}_{\mu}(\Lambda^*_{\mu})$ and close to 1 if $\ln N \geq (1 + o_n(1))\mathbb{E}_{\mu}(\Lambda^*_{\mu})$. The main condition is that the parameter $\beta(\mu) = \operatorname{Var}_{\mu}(\Lambda^*_{\mu})/(\mathbb{E}_{\mu}(\Lambda^*_{\mu}))^2$ should be small. The talk is based on joint works with S. Brazitikos and M. Pafis.

Haddad, Julián (University of Seville, Spain)

Title: The L_p Higher-order Petty projection inequality

Abstract: The higher-order projection body was defined very recently as the body whose support function is the directional derivative of the higher-order covariogram function, which was defined by Schneider in the 70's. I will present an L^p analog of the higher-order projection body and the corresponding Petty projection inequality. The characterization of the equality case and the limiting behaviour as $p \to \infty$ recover apparently unrelated results. (joint work with D. Langharst, E. Putterman, M. Roydson and D. Ye)

Hofstätter, Georg (Friedrich Schiller University Jena, Germany)

Title: Equivariant Valuations on Convex Functions

Abstract: In the affine geometry of convex bodies, many fundamental constructions like the difference body map and the projection body map can be characterized by continuity, a valuation property, translation-invariance and their equi- or contravariance, respectively, with respect to volume preserving linear maps. In this talk, we transfer this approach to the theory of valuations on convex functions and give a characterization of all continuous and dually epi-translation invariant valuations on finite convex functions with values in the same space, which are SL_n equi- or contravariant, respectively, thereby generalizing theorems by M. Ludwig. This is joint work with J. Knoerr.

Hug, Daniel (Karlsruhe Institute of Technology, Germany)

Title: Equality cases in the Alexandrov–Fenchel inequality for a class of convex bodies

Abstract: Mixed volumes in *n*-dimensional Euclidean space are functionals of *n*-tuples of convex bodies $K, L, C_1, \ldots, C_{n-2}$. The Alexandrov–Fenchel inequalities (AFI) are fundamental inequalities between mixed volumes of convex bodies, which cover as very special cases many important inequalities between basic geometric functionals. Up to now a complete characterization of the equality cases in the Alexandrov–Fenchel equality is missing. Major recent progress was made by Yair Shenfeld and Ramon von Handel, in particular they resolve the problem in the cases where C_1, \ldots, C_{n-2} are polytopes, zonoids or smooth bodies. We introduce a new class of convex bodies, which includes polytopes and zonoids as special examples, and establish a characterization theorem for the members of this class. Based on this and Shenfeld and van Handel's contribution, we extend their study of the extremals of the AFI.

(based on joint work with P.A. Reichert)

Knoerr, Jonas (TU Wien, Austria)

Title: Monge-Ampère operators and valuations

Abstract: Monge-Ampère-type operators play an important role in many problems in analysis and geometry. Many of these operators can naturally be considered as measurevalued valuations on convex functions, and consequently, they have found a number of applications in the construction of invariant valuations on convex bodies and functions. In this talk, I will present a characterization of a certain class of measure-valued valuations and different descriptions of these functionals in terms of mixed Monge-Ampère operators and differential forms.

Koldobsky, Alexander (University of Missouri, Columbia, US)

Title: Comparison problems for the Radon transform

Abstract: Given two non-negative functions such that the Radon transform of one of them is pointwise smaller, does it follow that the L^p -norm of this function is smaller for a given p > 1? We consider this problem for the classical and spherical Radon transforms. In both cases we point out classes of functions for which the answer is affirmative, and show that in general the answer is negative if the functions do not belong to these classes. The results are in the spirit of the solution of the Busemann-Petty problem from convex geometry, and the classes of functions that we introduce generalize the class of intersection bodies introduced by Lutwak. This is joint work with Michael Roysdon and Artem Zvavitch.

Kotrbatý, Jan (Goethe University Frankfurt, Germany)

Title: Inequalities for higher-rank mixed volumes

Abstract: Going back to H. Minkowski, the concept of mixed volume assigns to a tuple of convex bodies in an *n*-dimensional vector space a natural numerical quantity which satisfies the fundamental Alexandrov–Fenchel inequality. In a joint work with Thomas Wannerer, we introduce an array of new numerical invariants, mixed volumes of rank k = 1, ..., n - 1, subsuming the classical mixed volume for k = 1. We prove that also the opposite boundary case k = n - 1 satisfies an Alexandrov–Fenchel-type inequality and that this new inequality sharpens, in turn, the classical Alexandrov–Fenchel inequality, when applied to lower-dimensional convex bodies.

Langharst, Dylan (Kent State University, US)

Title: Higher-Order Difference and Projection Bodies, and their associated Inequalities

Abstract: The volumetric bounds on the difference body of a convex body (encoded in the Brunn-Minkowski and Rogers-Shephard inequalities) are measurements of asymmetry of a convex body. Schneider generalized the difference body to higher-order and established the associated higher-order Rogers-Shephard inequality. As a tool of this work, Schneider generalized the covariogram of a convex body. Continuing on his work, we use the fact that the classical covariogram is related to the projection and polar projection bodies of a convex body and prove the analogous higher-order connection. We thus are able to define higher-order radial mean bodies and polar projection bodies. The associated higher-order Zhang-and-Petty-type inequalities will be shown. If time permits, we discuss applications to sharp higher-order affine Sobolev inequalities. Joint work with J. Haddad, E. Putterman, M. Roysdon and D. Ye.

Li, Ai-Jun (Zhejiang University of Science and Technology, China)

Title: Some geometric and functional inequalities related to lower dimensional subspaces

Abstract: In this talk, we consider several functions related to lower dimensional projections and intersections of convex bodies. Some properties like the bound, the affine nature, the continuity, and the monotonicity are provided. Moreover, a Khinchine type inequality for m-dimensional subspaces is established. As an application, we establish a generation of Zhang's inequality.

Li, Jin (Shanghai University, China)

Title: The Legendre transform and dually epi-translation contravariant valuations

Abstract: Epi-translations or dual epi-translations play important roles in the recently developing theory of valuations on convex functions. In this talk, I will present a characterization of the Legendre transform with dual epi-translations, and equivalently, a characterization of the identity map with epi-translations.

Lin, Youjiang (Chongqing Technology and Business University, China)

Title: Projection bodies in spherical and hyperbolic spaces

Abstract: Using gnomonic projection and Poincare model, we first define the spherical projection body and hyperbolic projection body in spherical space \mathbb{S}^n and hyperbolic space \mathbb{H}^n , then define the spherical Steiner symmetrization and hyperbolic Steiner symmetrization, finally prove the spherical projection inequality and hyperbolic projection inequality.

Mathis, Léo (Goethe University Frankfurt, Germany)

Title: Expectation of random submanifold with zonoids

Abstract: In a joint work with Michele Stecconi, we show how to evaluate expectation of volumes of random submanifold when they are given as zero sets of "nice enough" random fields. I will explain how this involves the construction of a zonoid in the cotangent space of the ambient manifold via a Kac-Rice formula and how this relates to the zonoid algebra.

Mui, Stephanie (Courant Institute of Mathematical Sciences, New York University, US)

Title: TBA

Abstract: TBA

Ortega-Moreno, Oscar (TU Wien, Austria)

Title: Fixed Points of Mean Section Operators

Abstract: In this talk, we study regularizing properties of convolution transforms on the unit sphere. More precisely, we show that they are determined by the mass distribution of the Laplacian of the kernel near the poles. As an application, we characterize fixed points of Minkowski valuations in a smooth neighborhood of the unit ball. This extends previous results to a larger class of Minkowski valuations, including the mean section operators. Moreover, we refine an important representation theorem for homogeneous Minkowski valuations by proving that their generating functions are locally Lipschitz outside the poles. This is joint work with Leo Brauner.

Paouris, Grigoris (Texas A&M University, College Station, US)

Title: Concentration on marginals of p-Schatten norms

Abstract: We investigate the tail estimates of the marginals of the uniform measure on the unit ball of the p-Schatten norm of matrices over the reals or complex numbers. We

establish sharp bounds for all directions and for all values of p. Based on joint work with Kavita Ramanan.

Rotem, Liran (Israel Institute of Technology, Haifa, Israel)

Title: Stability and equality cases for the Gaussian (B) inequality

Abstract: The (B) inequality is a property of the Gaussian measure, proved by Cordero-Erasuquin, Fradelizi and Maurey. It shows that for symmetric convex bodies one may improve the standard log-concavity estimate that follows from the Prekopa-Leindler inequality.

The original proof of the inequality involves smooth approximations and therefore gives no information about the equality cases. In this talk we will settle this issue by proving a stability result: If the inequality is "almost" an equality, the body involved should be "close" to either the whole space or a lower dimensional body.

Based on joint work with Orli Herscovici, Galyna Livshyts and Sasha Volberg.

Saroglou, Christos (University of Ioannina, Greece)

Title: On a j-Santaló Conjecture

Abstract: Let $k \ge 2$ be an integer. In the spirit of Kolesnikov-Werner, for each $j \in \{2, \ldots, k\}$, we conjecture a sharp Santaló type inequality (we call it *j*-Santaló conjecture) for many sets (or more generally for many functions), which we are able to confirm in some cases, including the case j = k and the unconditional case. Interestingly, the extremals of this family of inequalities are tuples of the l_j^n -ball. Our results also strengthen one of the main results of Kolesnikov-Werner, which corresponds to the case j = 2. All members of the family of our conjectured inequalities can be interpreted as generalizations of the classical Santaló inequality. Related, we discuss an analogue of a conjecture due to K. Ball in the multi-entry setting and establish a connection to the *j*-Santaló conjecture.

Schneider, Rolf (University of Freiburg, Germany)

Title: Expected valuations of random zonotopes

Abstract: Let X_1, \ldots, X_n be stochastically independent, identically distributed random points in \mathbb{R}^d . Their convex hull is a random polytope, thoroughly investigated in stochastic geometry. Instead, we propose to study the Minkowski sum of the segments connecting the X_i with the origin, resulting in a random zonotope Z_n . Let φ be a translation invariant, continuous valuation, homogeneous of degree $j \in \{1, \ldots, d\}$. For example, φ could be the *j*th intrinsic volume. Then $\varphi(Z_n)$ is a random variable. Under the assumption $\mathbb{E}||X_i|| < \infty$ we can determine its expectation, and assuming $\mathbb{E}||X_i||^2 < \infty$ and some further mild conditions on the probability distribution and the valuation, we obtain a central limit theorem, as $n \to \infty$.

Semenov, Vadim (New York University, US)

Title: The Uniqueness of the Gauss Image Measure

Abstract: The Gauss Image problem is a generalization to the question originally posed by Aleksandrov who studied the existence and uniqueness of the convex body with the prescribed Aleksandrov's integral curvature. In this talk, we are going to address the uniqueness of the solution to the Gauss Image Problem. Firstly, we will address how the uniqueness of the solution neatly follows from the Hopf Theorem under additional smooth assumptions. After which, we are going to talk about the strategy of the proof for the general case of the problem which involves new measure-theoretic concepts for multivalued maps and the newly established variational Lipschitz behavior of the radial Gauss Image map.

Tatarko, Kateryna (University of Waterloo, Canada)

Title: Reverse isoperimetric problems under curvature constraints

Abstract: In this talk, we will discuss the reverse isoperimetric problems in the class of λ -convex bodies, i.e., convex bodies with curvature at each point of their boundary bounded below by some positive parameter λ . This is a joint work with Kostiantyn Drach.

Ulivelli, Jacopo (Università La Sapienza, Rome, Italy)

Title: Entire Monge-Ampère equations and weighted Minkowski problems

Abstract: We present a geometric approach to investigate Monge-Ampère equations depending on the Fenchel-Legendre transform of their solution. The core idea is to translate solutions of a suitable Minkowski problem (on the sphere) into solutions of the equation at hand, expanding some classical results of Pogorelov and Bakelman.

Wannerer, Thomas (Friedrich Schiller University Jena, Germany)

Title: On a generalization of the Alexandrov-Fenchel inequality

Abstract: The Alexandrov-Fenchel inequality is one the most powerful and versatile inequalities in the Brunn-Minkowski theory of convex bodies. In the 1970s, Khovanskii and Teissier discovered a remarkable connection between the Alexandrov-Fenchel inequality and the Hodge- Riemann bilinear relations. Ross and Toma have recently discovered a generalization of the Hodge-Riemann bilinear relations to Schur polynomials in Kähler forms. In this talk, we outline an alternative approach to this result via the theory of Lorentzian polynomials, recently introduced by Bränden and Huh. Our approach works in various situations and yields in particular a generalization of the Alexandrov-Fenchel inequality.

Based on joint work with Julius Ross and Hendrik Süss.

Wyczesany, Katarzyna (Carnegie Mellon University, Pittsburg, US)

Title: Stability of polydisc slicing

Abstract: We prove a dimension-free stability result for polydisc slicing. Intriguingly, compared to the real case, there is an additional asymptotic maximiser. In addition to Fourier-analytic bounds developed by Oleszkiewicz and Pelczynski, we crucially rely on a self-improving feature of polydisc slicing, established via probabilistic arguments. This is joint work with Nathaniel Glover and Tomasz Tkocz.

Yepes Nicolás, Jesús (University of Murcia, Spain)

Title: On complemented Brunn-Minkowski type inequalities

Abstract: A measure μ on \mathbb{R}^n is said to be q-concave if it satisfies a q-Brunn-Minkowski inequality, namely,

$$\mu((1-\lambda)A + \lambda B) \ge ((1-\lambda)\mu(A)^q + \lambda\mu(B)^q)^{1/q}$$

for all measurable sets $A, B \subset \mathbb{R}^n$ with $\mu(A)\mu(B) > 0$ such that $(1 - \lambda)A + \lambda B$ is also measurable, and all $\lambda \in (0, 1)$.

Following the duality of concave and convex functions, it is natural to wonder about a *q*-complemented Brunn-Minkowski inequality, i.e., whether

$$\mu \big(\mathbb{R}^n \setminus \big((1-\lambda)A + \lambda B \big) \big) \le \big((1-\lambda)\mu \big(\mathbb{R}^n \setminus A \big)^q + \lambda \mu \big(\mathbb{R}^n \setminus B \big)^q \big)^{1/q},$$

provided that $\mu(\mathbb{R}^n \setminus A), \mu(\mathbb{R}^n \setminus B) < +\infty.$

When $(\mu \text{ is finite and}) q = 1$ both conditions above are trivially equivalent, but this equivalence is no longer true in general for other values of q. However, Milman and Rotem in 2014 showed that under certain assumptions of concavity and homogeneity for the density of μ such an inequality holds. In particular, the restriction of the Lebesgue measure vol (\cdot) to a convex cone C (which is its support) satisfies the latter inequality for q = 1/n and any $A, B \subset C$ with vol $(C \setminus A)$, vol $(C \setminus B) < +\infty$. This case was later studied also by Schneider in 2018, who gave a different proof and characterized its equality case when A and B are convex.

In this talk we will discuss about different functional and geometric forms of complemented Brunn-Minkowski type inequalities for certain absolutely continuous measures on \mathbb{R}^n containing, among others, both the volume and the standard Gaussian measure γ_n . In particular, we will show the connection between this family of complemented Brunn-Minkowski inequalities and that of dual Brunn-Minkowski inequalities (involving the radial sum).

This is about joint work in progress with A. Zvavitch.

Organization and sponsors

Scientific committee

Andrea Colesanti (University of Florence), Monika Ludwig (TU Wien), Gaoyong Zhang (Courant Institute of Mathematical Sciences, New York University).

Organizing committee

Gabriele Bianchi (University of Florence), Paolo Gronchi (University of Florence), Ilaria Lucardesi (University of Florence).

Sponsors

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