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Regularity properties of weak solutions to a diffusive shallow medium equation

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joint work with Thomas Singer[2]

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We discuss regularity properties of weak solutions to the doubly nonlinear parabolic equation

$$\partial_t u - \nabla \cdot ((u-z)^\alpha |\nabla u|^{p-2}\nabla u) = f \quad \text{in } \Omega_T := \Omega \times (0,T),$$

where $\Omega \subset \mathbb{R}^n$ is an open bounded set, $z : \Omega \to \mathbb{R}$ and $f : \Omega_T \to \mathbb{R}$ are given sufficiently regular functions, and the parameters $\alpha > 0$ and $p > 1$ satisfy $\alpha + p > 2$. In the range $p < 2$, the equation has applications in models of shallow water dynamics, and for $p > 2$ the equation has been used to model the dynamics of glaciers. Regardless of the range of p, z represents the elevation of the land on top of which the water or ice is moving. The value of u is the height of the medium and f is a source term which can represent snow in the case of glaciers, and rainfall, infiltration or evaporation in the shallow water setting.

We present the natural definition of weak solutions allowing us to obtain energy estimates which are combined with Sobolev inequaties to prove local boundedness.[1] In the range $p > 2$ we can also conclude local Hölder continuity.[2]

References

[1] T. Singer and M. Vestberg. Local Boundedness of Weak Solutions to the Diffusive Wave Approximation of the Shallow Water Equations, *Journal of Differential Equations*, https://doi.org/10.1016/j.jde.2018.08.051, in press.

[2] T. Singer and M. Vestberg. Local Hölder Continuity of Weak Solutions to a Diffusive Shallow Medium Equation, Manuscript.