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Nonparametric minimal surfaces with thin obstacles

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I will discuss some recent progress [1] on a classical problem in the calculus of variations, concerning the equilibrium configurations of a minimal surface on a thin obstacle:

$$\min_{v\in\mathcal{A}_g} \int_{B_1} \sqrt{1+|\nabla v|^2}\,dx \tag{0.1}$$

in the class $\mathcal{A}_g := \left\{ v \in g|_{B_1} + W_0^{1,\infty}(B_1) : \, v|_{B_1\cap\{x_{n+1}=0\}} \geq 0, v(x', x_{n+1}) = v(x', -x_{n+1}) \right\}$, where $g : B_1 \subseteq \mathbb{R}^{n+1} \to \mathbb{R}$ is a given boundary data.

We are interested in discussing the regularity of the solution u and of the boundary (in the relative topology of B_1') of the contact set $\{(x',0) \in B_1' : u(x',0) = 0\}$, also called the *free boundary*,

$$\Gamma(u) := \partial_{B_1'}\{(x',0) \in B_1' : u(x',0) = 0\},$$

extending several previous results due to the works by Nitsche [5], Giusti [3, 4], Frehse [2] and Ural'tseva [6].

References

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