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Global and cocycle attractors for non-autonomous scalar reaction-diffusion equations

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joint work with Tomás Caraballo[2], José A. Langa[2], and Rafael Obaya[1]

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In this talk we consider the skew-product semiflow induced by the mild solutions of a family of scalar linear-dissipative parabolic problems over a minimal and uniquely ergodic flow $(P,\cdot,\mathbb{R})$, given for each $p \in P$ by

$$\begin{cases} \dfrac{\partial y}{\partial t} = \Delta y + h(p\cdot t,x)\,y + g(p\cdot t,x,y), & t>0,\ x\in U,\\ By := \alpha(x)\,y + \dfrac{\partial y}{\partial n} = 0, & t>0,\ x\in \partial U. \end{cases}$$

The structure of the global and cocycle attractors in the case that λ_P, the upper Lyapunov exponent of the associated linear family, is different from zero has been investigated in Cardoso et al. [2]. We now study the same problem when $\lambda_P = 0$ to show that these attractors exhibit a rich dynamics that frequently contains ingredients of high complexity.

Basically, two different types of attractors can appear, depending on whether the linear equations have a bounded or an unbounded associated real cocycle. In the first case (e.g. in periodic equations), the structure of the attractor is simple, whereas in the second case (which occurs in aperiodic equations), the attractor is a pinched set with a complicated structure. We describe situations when the attractor is chaotic in measure in the sense of Li-Yorke. Besides, we obtain a non-autonomous discontinuous pitchfork bifurcation scenario for concave equations.

References

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[2] C.A. Cardoso, J.A. Langa, R. Obaya, Characterization of cocycle attractors for nonautonomous reaction-diffusion equations, *Internat. J. Bifur. Chaos*, **26** (8): id.1650135-263, 2016.