Workshop on dynamical systems, calculus of variations and control
Florence, 8th-9th November 2018

Some fourth-order models for suspension bridges with multiple spans: towards optimizing stability

Maurizio Garrione[1,2]*

joint work with Filippo Gazzola[1]

[1]Department of Mathematics, Polytechnic of Milan

[2]Department of Mathematics, Polytechnic of Milan

maurizio.garrione@polimi.it (*), filippo.gazzola@polimi.it

We discuss some fourth-order models with "multi-point"-conditions having the aim of describing the oscillations of a suspension bridge with four internal piers. The final target is to address some instances of a degenerate plate-type model like

$$\begin{cases} u_{tt} + u_{xxxx} + f(u, \theta) = 0 \\ \theta_{tt} - \theta_{xx} + g(u, \theta) = 0\,, \end{cases} \qquad u = u(x,t),\ \theta = \theta(x,t),$$

with boundary and junction conditions

$$u(-\pi, t) = u(\pi, t) = \theta(-\pi, t) = \theta(\pi, t) = 0 \qquad t \geq 0\,,$$

$$u(-b\pi, t) = u(a\pi, t) = \theta(-b\pi, t) = \theta(a\pi, t) = 0 \qquad t \geq 0\,,$$

describing the oscillations of a structure composed by a central beam of length 2π (u being its vertical displacement) and by a continuum of cross sections allowed to rotate around it (with angular displacement θ), with piers in correspondence of the cross sections $x = -b\pi$, $x = a\pi$, $0 < a, b < 1$. First, we present complete spectral results for the associated linear stationary problems, explicitly underlining the possible losses of regularity caused by the presence of the piers. We then investigate the optimal positions of the piers in terms of suitable notions of (linear and nonlinear) stability, using both analytical tools (such as Floquet theory) and numerical simulations.