Some remarks on the multiplicity of Ground States for the scalar curvature equation.

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joint work with Francesca DALBONO[2], and Andrea SFECCI[1]

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In this talk we discuss the problem of existence and multiplicity of radial ground states with fast decay (GS for short) for

$$\Delta u + [1 + \epsilon k(|x|)]u^{\frac{n+2}{n-2}} = 0$$

where $x \in \mathbb{R}^n$, $n \geq 3$, $k \in C^1$, $k(|x|) \in [0,1]$, $\epsilon > 0$ small. Nowadays several different conditions sufficient for the existence of GS are available in literature. Further, if k has a unique critical point and it is a maximum the GS is unique, see [3]. On the other side if the unique critical point is a minimum (and some other conditions are fulfilled) a large number of GS are found, if $\epsilon > 0$ is small enough, see [1]. A similar result was obtained in [2] replacing $[1 + \epsilon k(|x|)]$ by a slowly varying function $k(|x|^\epsilon)$.

Our purpose is to give a constructive argument which enable us to re-prove the result in [1] but giving an estimate on how small ϵ should be. In fact ϵ need not to be very small, e.g. we have at least k GS for $\epsilon < \frac{1}{k}$ if $n = 4$.

References

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