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Positive periodic solutions to an indefinite Minkowski-curvature equation

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We deal with the indefinite Minkowski-curvature equation

$$\left(\frac{u'}{\sqrt{1-(u')^2}}\right)' + \lambda a(t) g(u) = 0,$$

where λ is a positive parameter, $a(t)$ is a T-periodic sign-changing weight function and $g\colon [0,+\infty[\, \to [0,+\infty[$ is a continuous function having superlinear growth at zero. We prove that for both $g(u) = u^p$, with $p > 1$, and $g(u) = u^p/(1+u^{p-q})$, with $0 \leq q \leq 1 < p$, the equation has no positive T-periodic solutions for λ close to zero and two positive T-periodic solutions (a "small" one and a "large" one) for λ large enough. Moreover, in both cases the "small" T-periodic solution is surrounded by a family of positive subharmonic solutions with arbitrarily large minimal period. The proof of the existence of T-periodic solutions relies on a recent extension of Mawhin's coincidence degree theory for locally compact operators in product of Banach spaces (cf. [2]), while subharmonic solutions are found by an application of the Poincaré–Birkhoff fixed point theorem, after a careful asymptotic analysis of the T-periodic solutions for $\lambda \to +\infty$.

References

[1] A. Boscaggin, G. Feltrin. Positive periodic solutions to an indefinite Minkowski-curvature equation, preprint (2018), arXiv:1805.06659.

[2] G. Feltrin, F. Zanolin. An application of coincidence degree theory to cyclic feedback type systems associated with nonlinear differential operators, *Topol. Methods Nonlinear Anal.*, 50 (2): 683–726, 2017.