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Sub-Finsler geometry for three dimensional contact structures.

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Let f, g two vector fields in $\mathbb{R}^3$ satisfying the *Lie algebra rank condition* (also known as *Hormander condition*)

$$\text{span}\{f, g, [f, g]\}(q) = \mathbb{R}^3 \ \forall q \in \mathbb{R}^3.$$

Consider the problem of minimising the functional

$$\min_{u_1, u_2} \int_0^1 |u_1(t)| + |u_2(t)| dt \tag{0.1}$$

under all the solution of the control system

$$\begin{cases} \dot{q} = u_1 f(q) + u_2 g(q) \\ q(0) = (0,0,0),\ q(1) = (x_f, y_f, z_f). \end{cases} \tag{0.2}$$

with (u_1, u_2) measurable real-valued L^1 functions.

The problem (0.1)-(0.2) induces a metric structure on $\mathbb{R}^3$, where the distance between two points q_0 and q_f is given by the infimum of the functional (0.1), among all solutions of (0.2) with endpoints q_0 and q_f.

In this talk, we will discuss the solutions of the problem (0.1)-(0.2) in the nilpotent case (see also [1]), and we will show some preliminary results on the generic case.

References

[1] D. Barilari, U. Boscain, E. Le Donne and M. Sigalotti. Sub-Finsler structures from the time-optimal control viewpoint for some nilpotent distributions *Journal of Dynamical and Control Systems*, 23 (3):547–575, 2017.