Workshop on dynamical systems, calculus of variations and control

Florence, 8th-9th November 2018

UNIVERSITÀ DEGLI STUDI FIRENZE

Last update November 6, 2018

SCHEDULE

Thursday 8th November	
14:15	BÖGELEIN
14:55	LIAO
15:35	Coffee Break
16:10	PÖTZSCHE
16:50	FRANCA
17:30	SANZ
	Social dinner
Friday 9th November	
9:00	ROCCA
9:40	SPADARO
10:20	Coffee Break
10:50	CATANIA
11:30	CHITTARO
12:10	GUERRA
12:50	Lunch
14:00	FELTRIN
14:40	GARRIONE
15:20	BOSCAGGIN
16:00	VESTBERG

Higher integrability for doubly nonlinear evolution equations

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In this talk we establish the higher integrability of the spatial gradient of weak solutions to doubly nonlinear evolution equations of the type

$$\partial_t\Big(|u|^{p-2}u\Big) - \operatorname{div}\Big(|Du|^{p-2}Du\Big) = \operatorname{div}\Big(|F|^{p-2}F\Big).$$

We prove that there exists $\epsilon > 0$ such that

$$|F| \in L^{p+\epsilon}_{loc} \quad \Rightarrow \quad |Du| \in L^{p+\epsilon}_{loc}.$$

Scattering parabolic solutions for the N-centre problem

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joint works with Walter Dambrosio[1], Duccio Papini[2] and Susanna Terracini[1]

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For the N-centre problem, both in the two dimensional and in the three dimensional Euclidean space, we prove the existence/multiplicity of entire parabolic trajectories having prescribed asymptotic directions. The proof relies on variational arguments (local minimization or critical point theory); level estimates, Morse index estimates and regularization techniques are used in order to rule out the possible occurrence of collisions.

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On the Euler–Voigt System in a 3D Bounded Domain

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joint work with Luigi C. Berselli[2]

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We consider the Euler–Voigt equations in a bounded domain as an approximation for the 3D Euler equations. We show that the solutions of the Voigt equations are global, do not smooth out the solutions and converge to the solutions of the Euler equations, hence they represent a good model. [1].

Bibliography

[1] Luigi C. Berselli and Davide Catania. On the Euler–Voigt System in a 3D Bounded Domain: Propagation of Singularities and Absence of the Boundary Layer, Submitted.

Sub-Finsler geometry for three dimensional contact structures.

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joint work with Fazia Harrache[1,2]

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Let f, g two vector fields in $\mathbb{R}^3$ satisfying the Lie algebra rank condition (also known as Hormander condition)

$$\mathrm{span}\{f, g, [f, g]\}(q) = \mathbb{R}^3 \ \forall q \in \mathbb{R}^3.$$

Consider the problem of minimising the functional

$$\min_{u_1,u_2} \int_0^1 |u_1(t)| + |u_2(t)| dt \tag{0.0.1}$$

under all the solution of the control system

$$\begin{cases} \dot{q} = u_1 f(q) + u_2 g(q) \\ q(0) = (0,0,0), \ q(1) = (x_f, y_f, z_f). \end{cases} \tag{0.0.2}$$

with (u_1, u_2) measurable real-valued L^1 functions.

The problem (0.0.1)-(0.0.2) induces a metric structure on $\mathbb{R}^3$, where the distance between two points q_0 and q_f is given by the infimum of the functional (0.0.1), among all solutions of (0.0.2) with endpoints q_0 and q_f.

In this talk, we will discuss the solutions of the problem (0.0.1)-(0.0.2) in the nilpotent case (see also [1]), and we will show some preliminary results on the generic case.

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Positive periodic solutions to an indefinite Minkowski-curvature equation

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joint work with Alberto Boscaggin[2]

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We deal with the indefinite Minkowski-curvature equation

$$\left(\frac{u'}{\sqrt{1-(u')^2}}\right)' + \lambda a(t) g(u) = 0,$$

where λ is a positive parameter, $a(t)$ is a T-periodic sign-changing weight function and $g\colon [0,+\infty[\, \to [0,+\infty[$ is a continuous function having superlinear growth at zero. We prove that for both $g(u)=u^p$, with $p>1$, and $g(u)=u^p/(1+u^{p-q})$, with $0\leq q\leq 1<p$, the equation has no positive T-periodic solutions for λ close to zero and two positive T-periodic solutions (a "small" one and a "large" one) for λ large enough. Moreover, in both cases the "small" T-periodic solution is surrounded by a family of positive subharmonic solutions with arbitrarily large minimal period. The proof of the existence of T-periodic solutions relies on a recent extension of Mawhin's coincidence degree theory for locally compact operators in product of Banach spaces (cf. [2]), while subharmonic solutions are found by an application of the Poincaré–Birkhoff fixed point theorem, after a careful asymptotic analysis of the T-periodic solutions for $\lambda \to +\infty$.

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Some remarks on the multiplicity of Ground States for the scalar curvature equation.

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joint work with Francesca DALBONO[2], and Andrea SFECCI[1]

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In this talk we discuss the problem of existence and multiplicity of radial ground states with fast decay (GS for short) for

$$\Delta u + [1 + \epsilon k(|x|)]u^{\frac{n+2}{n-2}} = 0$$

where $x \in \mathbb{R}^n$, $n \geq 3$, $k \in C^1$, $k(|x|) \in [0, 1]$, $\epsilon > 0$ small. Nowadays several different conditions sufficient for the existence of GS are available in literature. Further, if k has a unique critical point and it is a maximum the GS is unique, see [3]. On the other side if the unique critical point is a minimum (and some other conditions are fulfilled) a large number of GS are found, if $\epsilon > 0$ is small enough, see [1]. A similar result was obtained in [2] replacing $[1 + \epsilon k(|x|)]$ by a slowly varying function $k(|x|^\epsilon)$.

Our purpose is to give a constructive argument which enable us to reprove the result in [1] but giving an estimate on how small ϵ should be. In fact ϵ need not to be very small, e.g. we have at least k GS for $\epsilon < \frac{1}{k}$ if $n = 4$.

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Some fourth-order models for suspension bridges with multiple spans: towards optimizing stability

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joint work with Filippo Gazzola[1]

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We discuss some fourth-order models with "multi-point"-conditions having the aim of describing the oscillations of a suspension bridge with four internal piers. The final target is to address some instances of a degenerate plate-type model like

$$\begin{cases} u_{tt} + u_{xxxx} + f(u,\theta) = 0 \\ \theta_{tt} - \theta_{xx} + g(u,\theta) = 0, \end{cases} \qquad u = u(x,t),\ \theta = \theta(x,t),$$

with boundary and junction conditions

$$u(-\pi,t) = u(\pi,t) = \theta(-\pi,t) = \theta(\pi,t) = 0 \qquad t \geq 0,$$

$$u(-b\pi,t) = u(a\pi,t) = \theta(-b\pi,t) = \theta(a\pi,t) = 0 \qquad t \geq 0,$$

describing the oscillations of a structure composed by a central beam of length 2π (u being its vertical displacement) and by a continuum of cross sections allowed to rotate around it (with angular displacement θ), with piers in correspondence of the cross sections $x = -b\pi$, $x = a\pi$, $0 < a, b < 1$. First, we present complete spectral results for the associated linear stationary problems, explicitly underlining the possible losses of regularity caused by the presence of the piers. We then investigate the optimal positions of the piers in terms of suitable notions of (linear and nonlinear) stability, using both analytical tools (such as Floquet theory) and numerical simulations.

On the transport of measures by controlled flows

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joint work with Andrey Sarychev[2]

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We consider a smooth control system with n-dimensional space state. For such a system, a natural extension of the notion of controllability is as follows: given two Borel probability measures in $\mathbb{R}^n$, is there a control such that the corresponding flow by the control system transports the first measure into the second?

Realistically, we can not expect to obtain exact controllability in the sense above. However, we discuss some conditions which imply approximate controllability in a weak sense.

A Wiener-type boundary estimate for degenerate and singular diffusion equations

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In this talk, I will present a boundary estimate for solutions to degenerate and singular diffusion equations of p-laplacian type in cylindrical domains. The estimate is given in terms of a Wiener-type integral, defined by a proper elliptic p-capacity.

Topological decoupling and linearization of nonautonomous evolution equations

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Topological linearization results typically require solution flows rather than merely semiflows. An exception occurs when the linearization fulfills spectral assumptions met e.g. for scalar reaction-diffusion equations. We employ tools from the geometric theory of nonautonomous dynamical systems in order to extend earlier work by Lu [1] to time-variant evolution equations under corresponding conditions on the Sacker-Sell spectrum of the linear part. Our abstract results are applied to nonautonomous reaction-diffusion and convection equations.

The talk is based on joint work with Evamaria Russ.

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[2] C. Pötzsche and E. Ruß. Topological decoupling and linearization of nonautonomous evolution equations. Discrete Contin. Dyn. Syst. (Series S) 9(4), 1235–1268, 2016

Optimal control and long-time behavior of solutions for a diffuse interface model of tumor growth

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We consider the problem of long-time behavior of solutions and optimal control for a diffuse interface model of tumor growth. The state equations couples a Cahn-Hilliard equation and a reaction-diffusion equation, which models the growth of a tumor in the presence of a nutrient and surrounded by host tissue. The introduction of drugs into the system through the nutrient serves to eliminate the tumor cells, hence, in this setting the control variable will act on the nutrient equation. Furthermore, we allow the objective functional to depend on a free time variable, which represents the unknown treatment time to be optimized. As a result, we obtain first order necessary optimality conditions for both the drug concentration and the treatment time. One of the main aim of the control problem is to realize in the best possible way a desired final distribution of the tumor cells which is expressed by a target function that can be taken as a stable configuration of the system, so that the tumor does not grow again once the treatment is completed. In view of this fact we consider here also the problem of long-time behavior of solutions.

This is a joint project with C. Cavaterra (University of Milan), A. Miranville (University of Poitiers), G. Schimperna (University of Pavia), H. Wu (Fudan University, Shanghai).

Global and cocycle attractors for non-autonomous scalar reaction-diffusion equations

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joint work with Tomás Caraballo[2], José A. Langa[2], and Rafael Obaya[1]

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In this talk we consider the skew-product semiflow induced by the mild solutions of a family of scalar linear-dissipative parabolic problems over a minimal and uniquely ergodic flow $(P, \cdot, \mathbb{R})$, given for each $p \in P$ by

$$\begin{cases} \dfrac{\partial y}{\partial t} = \Delta y + h(p{\cdot}t, x)\, y + g(p{\cdot}t, x, y)\,, \quad t > 0\,,\ x \in U, \\ By := \alpha(x)\, y + \dfrac{\partial y}{\partial n} = 0\,, \quad t > 0\,,\ x \in \partial U. \end{cases}$$

The structure of the global and cocycle attractors in the case that λ_P, the upper Lyapunov exponent of the associated linear family, is different from zero has been investigated in Cardoso et al. [2]. We now study the same problem when $\lambda_P = 0$ to show that these attractors exhibit a rich dynamics that frequently contains ingredients of high complexity.

Basically, two different types of attractors can appear, depending on whether the linear equations have a bounded or an unbounded associated real cocycle. In the first case (e.g. in periodic equations), the structure of the attractor is simple, whereas in the second case (which occurs in aperiodic equations), the attractor is a pinched set with a complicated structure. We describe situations when the attractor is chaotic in measure in the sense of Li-Yorke. Besides, we obtain a non-autonomous discontinuous pitchfork bifurcation scenario for concave equations.

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Nonparametric minimal surfaces with thin obstacles

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joint work with Matteo Focardi[2]

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I will discuss some recent progress [1] on a classical problem in the calculus of variations, concerning the equilibrium configurations of a minimal surface on a thin obstacle:

$$\min_{v \in \mathcal{A}_g} \int_{B_1} \sqrt{1 + |\nabla v|^2}\, dx \tag{0.0.3}$$

in the class $\mathcal{A}_g := \left\{ v \in g|_{B_1} + W_0^{1,\infty}(B_1) : \ v|_{B_1 \cap \{x_{n+1}=0\}} \geq 0, v(x', x_{n+1}) = v(x', -x_{n+1}) \right\}$, where $g : B_1 \subseteq \mathbb{R}^{n+1} \to \mathbb{R}$ is a given boundary data.

We are interested in discussing the regularity of the solution u and of the boundary (in the relative topology of B_1') of the contact set $\{(x', 0) \in B_1' : u(x', 0) = 0\}$, also called the free boundary,

$$\Gamma(u) := \partial_{B_1'} \{(x', 0) \in B_1' : u(x', 0) = 0\},$$

extending several previous results due to the works by Nitsche [5], Giusti [3, 4], Frehse [2] and Ural'tseva [6].

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Regularity properties of weak solutions to a diffusive shallow medium equation

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joint work with Thomas Singer[2]

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We discuss regularity properties of weak solutions to the doubly nonlinear parabolic equation

$$\partial_t u - \nabla \cdot ((u-z)^{\alpha} |\nabla u|^{p-2} \nabla u) = f \quad \text{in } \Omega_T := \Omega \times (0, T),$$

where $\Omega \subset \mathbb{R}^n$ is an open bounded set, $z : \Omega \to \mathbb{R}$ and $f : \Omega_T \to \mathbb{R}$ are given sufficiently regular functions, and the parameters $\alpha > 0$ and $p > 1$ satisfy $\alpha + p > 2$. In the range $p < 2$, the equation has applications in models of shallow water dynamics, and for $p > 2$ the equation has been used to model the dynamics of glaciers. Regardless of the range of p, z represents the elevation of the land on top of which the water or ice is moving. The value of u is the height of the medium and f is a source term which can represent snow in the case of glaciers, and rainfall, infiltration or evaporation in the shallow water setting.

We present the natural definition of weak solutions allowing us to obtain energy estimates which are combined with Sobolev inequaties to prove local boundedness.[1] In the range $p > 2$ we can also conclude local Hölder continuity.[2]

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